Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Thursday 22 May 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

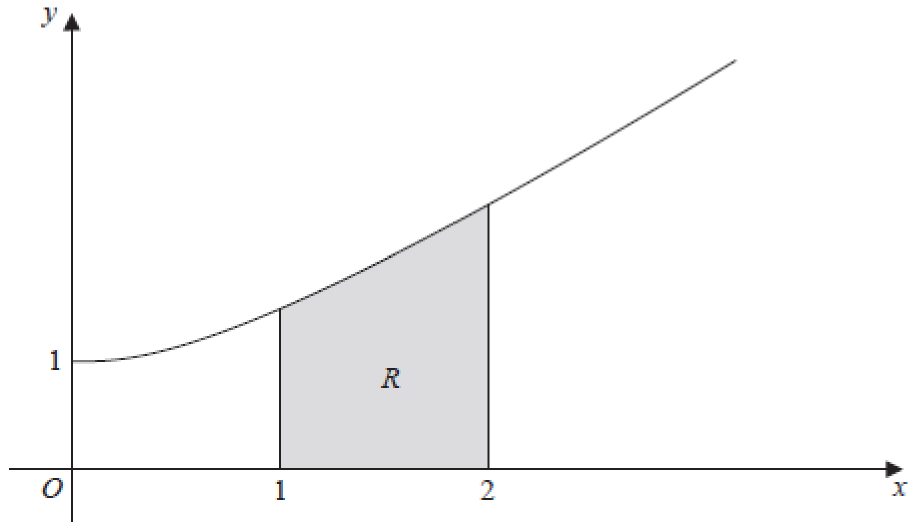

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geq 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$.

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414		1.803	2.016	2.236

(*a*) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

2.

$$f(x) = 2x^3 - 7x^2 + 4x + 4.$$

(*a*) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(*b*) Factorise $f(x)$ completely.

(4)

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3. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 - 3x)^6$, giving each term in its simplest form.

(4)

(*b*) Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$\left(1+\frac{x}{2}\right)(2-3x)^6.$$

(3)

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4. Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6}+\frac{1}{3x^2}\right) \mathrm{d}x,$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

5.

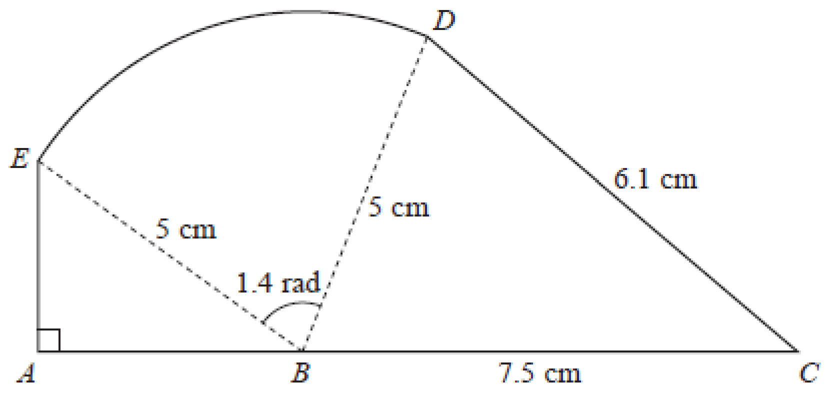

Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with $BC = 7.5$ cm.

Angle $EAB = \frac{\pi}{2}$ radians, angle $EBD = 1.4$ radians and $CD = 6.1$ cm.

(*a*) Find, in cm^2, the area of the sector *BDE*.

(2)

(*b*) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(*c*) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

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6. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is S_∞.

(*a*) Find the value of S_∞.

(2)

The sum to *N* terms of the series is S_N.

(*b*) Find, to 1 decimal place, the value of S_{12}.

(2)

(*c*) Find the smallest value of *N*, for which $S_\infty - S_N < 0.5$.

(4)

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7. (i) Solve, for $0 \leq \theta < 360°$, the equation $9 \sin(\theta + 60°) = 4$, giving your answers to 1 decimal place. You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leq x < \pi$, the equation $2 \tan x - 3 \sin x = 0$, giving your answers to 2 decimal places where appropriate.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

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8. (*a*) Sketch the graph of

$$y = 3^x, x \in \mathbb{R},$$

showing the coordinates of any points at which the graph crosses the axes.

(2)

(*b*) Use algebra to solve the equation $3^{2x} - 9(3^x) + 18 = 0$, giving your answers to 2 decimal places where appropriate.

(5)

9.

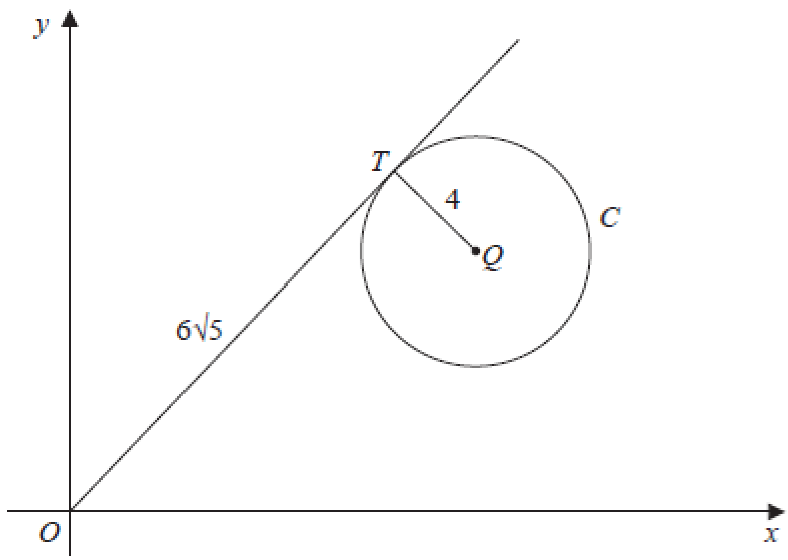


Figure 3

Figure 3 shows a circle C with centre Q and radius 4 and the point T which lies on C. The tangent to C at the point T passes through the origin O and $OT = 6\sqrt{5}$.

Given that the coordinates of Q are $(11, k)$, where k is a positive constant,

(*a*) find the exact value of k,

(3)

(*b*) find an equation for C.

(2)

10.

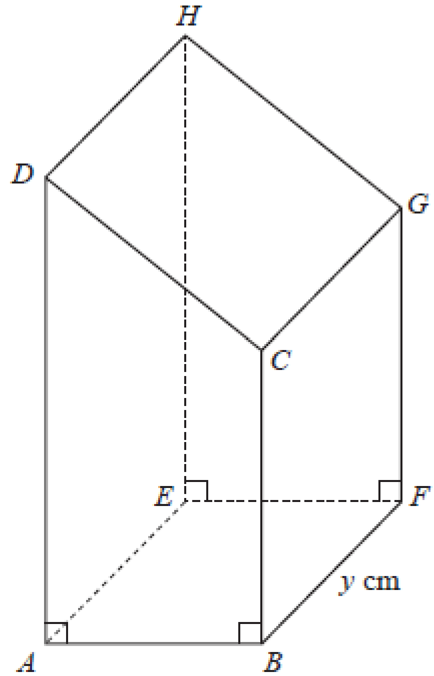


Figure 4

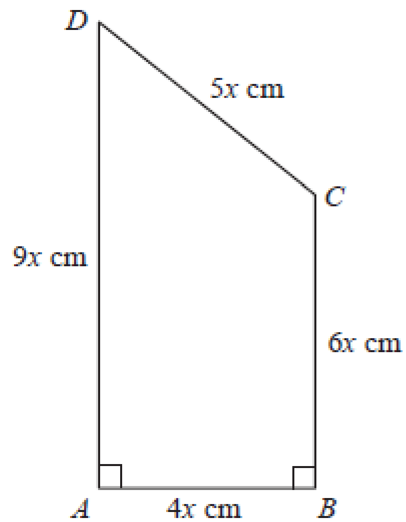


Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.

The angle $DAB = 90°$ and the angle $ABC = 90°$. The volume of the letter box is 9600 cm^3.

(*a*) Show that $y = \dfrac{320}{x^2}$.

(2)

(*b*) Hence show that the surface area of the letter box, S cm^2, is given by $S = 60x^2 + \dfrac{7680}{x}$.

(4)

(*c*) Use calculus to find the minimum value of S.

(6)

(*d*) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme			Marks
1.(a)	<table><tr><td>x</td><td>1</td><td>1.25</td><td>1.5</td><td>1.75</td><td>2</td></tr><tr><td>y</td><td>1.414</td><td>**1.601**</td><td>1.803</td><td>2.016</td><td>2.236</td></tr></table>			
	$\{\text{At } x = 1.25,\}\ y = 1.601$ (only)	1.601 **(May not be in the table and can score if seen as part of their working in (b))**		B1 cao
				[1]
(b)	$\frac{1}{2} \times 0.25; \times \underline{\left\{1.414 + 2.236 + 2\left(\text{their } 1.601 + 1.803 + 2.016\right)\right\}}$			B1; M1 A1ft
	B1; for using $\frac{1}{2} \times 0.25$ or $\frac{1}{8}$ or equivalent.	M1: Structure of $\underline{\{\ldots\ldots\ldots\ldots\ldots\}}$	A1ft: for the correct expression as shown following through candidate's y value found in part (a).	
	M1 requires the correct structure for the y values. It needs to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2(…..) bracket this may be regarded as a slip and the M mark can be allowed (nb: an extra repeated term, however, forfeits the M mark). M0 if any values used are x values instead of y values. A1ft: for the correct underlined expression as shown following through candidate's y value found in part (a). Bracketing mistakes: e.g. $\left(\frac{1}{2} \times \frac{1}{4}\right)(1.414 + 2.236) + 2\left(\text{their } 1.601 + 1.803 + 2.016\right)(= 11.29625)$ $\left(\frac{1}{2} \times \frac{1}{4}\right)1.414 + 2.236 + 2\left(\text{their } 1.601 + 1.803 + 2.016\right)(= 13.25275)$ Both score B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks could be given). **Alternative:** Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{8}(1.414 + 1.601) + \frac{1}{8}(1.601 + 1.803) + \frac{1}{8}(1.803 + 2.016) + \frac{1}{8}(2.016 + 2.236)\right]$ B1 for $\frac{1}{8}$ (aef), M1 for correct structure, 1st A1ft for correct expression, ft their 1.601			
	$\{= \tfrac{1}{8}(14.49)\} = 1.81125$	1.81 or awrt 1.81		A1
	Correct answer <u>only</u> in (b) scores no marks **If required accuracy is not seen in (a), full marks can still be scored in (b) (e.g. uses 1.6)**			
				[4]
				Total 5

Question Number	Scheme		Marks
	If there is no labelling, mark (a) and (b) in that order		
	$\mathrm{f}(x) = 2x^3 - 7x^2 + 4x + 4$		
2.(a)	$\mathrm{f}(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$	Attempts $\mathrm{f}(2)$ or $\mathrm{f}(-2)$	M1
	$= 0$, and so $(x-2)$ is a factor.	$\mathrm{f}(\mathbf{2}) = 0$ with no sign or substitution errors ($2(2)^3 - 7(2)^2 + 4(2) + 4 = 0$ is sufficient) **and for conclusion.** Stating "hence factor" or "it is a factor" or a "tick" or "QED" or "no remainder" or "as required" are fine for the conclusion **but not = 0 just underlined and not hence (2 or f(2)) is a factor**. Note also that a conclusion can be implied from a preamble, eg: "If $\mathrm{f}(2) = 0$, $(x-2)$ is a factor…."	A1
	Note: Long division scores no marks in part (a). The factor theorem is required.		
			[2]
(b)	$\mathrm{f}(x) = \{(x-2)\}(2x^2 - 3x - 2)$	M1: Attempts long division by $(x-2)$ or other method using $(x-2)$, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection."	M1 A1
		A1: $(2x^2 - 3x - 2)$	
	$= (x-2)(x-2)(2x+1)$ or $(x-2)^2(2x+1)$ or equivalent e.g. $= 2(x-2)(x-2)(x+\frac{1}{2})$ or $2(x-2)^2(x+\frac{1}{2})$	**d**M1: Factorises a 3 term quadratic. (see rule for factorising a quadratic in the General Principles for Core Maths Marking). **This is dependent** on the previous method mark being awarded but there must have been no remainder. Allow an attempt to solve the quadratic to determine the **factors**.	**d**M1 A1
		A1: cao – needs all three factors **on one line**. Ignore following work (such as a solution to a quadratic equation.)	
	Note $= (x-2)(\frac{1}{2}x - 1)(4x+2)$ would lose the last mark as it is not **fully** factorised		
	For correct answers only award full marks in (b)		
			[4]
			Total 6

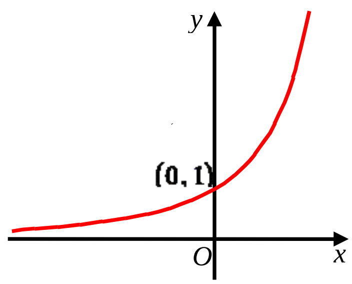
Question Number	Scheme		Marks
3. (a)	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\{(2-3x)^6\} = (2)^6 + \underline{{}^6C_1}(2)^5(-3\underline{x}) + \underline{{}^6C_2}(2)^4(-3\underline{x)^2} + ...$		<u>M1</u>
	M1: $\left({}^6C_1 \times ... \times x\right)$ or $\left({}^6C_2 \times ... \times x^2\right)$. For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. Ignore any extra terms.		
	${}^6C_1 2^5 - 3x + {}^6C_2 2^4 - 3x^2 + ...$ Scores M0 unless later work implies a correct method		
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1
			[4]
(a) Way 2	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\left(1 - \frac{3}{2}x\right)^6 = 1 + \underline{{}^6C_1}\left(\frac{-3}{2}\underline{x}\right) + \underline{{}^6C_2}\left(\frac{-3}{2}\underline{x}\right)^{\underline{2}} + ...$	M1: $\left({}^6C_1 \times ... \times x\right)$ or $\left({}^6C_2 \times ... \times x^2\right)$. For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. Ignore any extra terms.	<u>M1</u>
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1
(b)	Candidate writes down $\left(1 + \frac{x}{2}\right) \times \left(\text{their part (a) answer, at least up to the term in } x\right)$. (Condone missing brackets) $\left(1 + \frac{x}{2}\right)(64 - 576x + ...)$ or $\left(1 + \frac{x}{2}\right)(64 - 576x + 2160x^2 + ...)$ or $\left(1 + \frac{x}{2}\right)64 - \left(1 + \frac{x}{2}\right)576x$ or $\left(1 + \frac{x}{2}\right)64 - \left(1 + \frac{x}{2}\right)576x + \left(1 + \frac{x}{2}\right)2160x^2$ or $64 + 32x, -576x - 288x^2,\ 2160x^2 + 1080x^3$ are fine.		M1
	$= 64 - 544x + 1872x^2 + ...$	A1: At least 2 terms correct as shown. (Allow $+ -544x$ here) A1: $64 - 544x + 1872x^2$ The terms can be "listed" rather than added. Ignore any extra terms.	A1A1
			[3]
			Total 7
	SC: If a candidate expands in descending powers of x, only the M marks are available		
	e.g. $\{(2-3x)^6\} = (-3x)^6 + \underline{{}^6C_1}(2)^2(-3\underline{x})^5 + \underline{{}^6C_2}(2)^2(-3\underline{x})^4 + ...$		

Question Number	Scheme		Marks
4.		M1: $x^n \to x^{n+1}$	
		A1: At least one of either $\frac{x^4}{6(4)}$ or $\frac{x^{-1}}{(3)(-1)}$.	
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$	A1: $\frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$ or equivalent. e.g. $\frac{\frac{x^4}{6}}{4}+\frac{\frac{x^{-1}}{3}}{-1}$ (they will lose the final mark if they cannot deal with this correctly)	M1A1A1
	Note that some candidates may change the function prior to integrating e.g. $\int \frac{x^3}{6}+\frac{1}{3x^2}\,\mathrm{d}x = \int 3x^5+6\,\mathrm{d}x$ **in which case allow the M1 if** $x^n \to x^{n+1}$ **for their changed function and allow the M1 for limits if scored**		
	$\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(\sqrt{3})^{-1}}{-1(3)}\right)-\left(\frac{(1)^4}{24}+\frac{(1)^{-1}}{-1(3)}\right)$		**d**M1
	2nd **d**M1: For using limits of $\sqrt{3}$ and 1 on an integrated expression and subtracting the correct way round. **The 2nd M1 is dependent on the 1st M1 being awarded.**		
	$= \left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{1}{9}\sqrt{3}$	$\frac{2}{3}-\frac{1}{9}\sqrt{3}$ or $a=\frac{2}{3}$ and $b=-\frac{1}{9}$. Allow equivalent fractions for a and/or b and 0.6 recurring and/or 0.1 recurring but do **not** allow $\frac{6-\sqrt{3}}{9}$	A1**cso**
	This final mark is cao and cso – there must have been no previous errors		
			Total 5
	Common Errors (Usually 3 out of 5)		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \int\left(\frac{x^3}{6}+3x^{-2}\right)\mathrm{d}x = \frac{x^4}{6(4)}+\frac{3x^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{3(\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{3(1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{3}{\sqrt{3}}\right)-\left(\frac{1}{24}+\frac{3}{-1}\right)=\frac{10}{3}-\sqrt{3}$ A0		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \int\left(\frac{x^3}{6}+(3x)^{-2}\right)\mathrm{d}x = \frac{x^4}{6(4)}+\frac{(3x)^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(3\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{(3\times 1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{\sqrt{3}}{9}$ A0 **Note this is the correct answer but follows incorrect work.**		

Question Number	Scheme		Marks
5.(a)	Area $BDE = \frac{1}{2}(5)^2(1.4)$	M1: Use of the correct formula or method for the area of the sector	M1A1
	$= 17.5\ (\text{cm}^2)$	A1: 17.5 oe	
			[2]
(b)	**Parts (b) and (c) can be marked together**		
	$6.1^2 = 5^2 + 7.5^2 - (2\times5\times7.5\cos DBC)$ or $\cos DBC = \frac{5^2 + 7.5^2 - 6.1^2}{2\times5\times7.5}$ (or equivalent)		M1
	M1: A correct statement involving the angle DBC		
	Angle $DBC = 0.943201...$	awrt 0.943	A1
	Note that work for (b) may be seen on the diagram or in part (c)		
			[2]
(c)	**Note that candidates may work in degrees in (c) (**Angle $DBC = 54.04....\text{degrees}$)		
	Area $CBD = \frac{1}{2}5(7.5)\sin(0.943)$		
	Angle $EBA = \pi - 1.4 - "0.943"$ **(Maybe seen on the diagram)**	Area $CBD = \frac{1}{2}5(7.5)\sin(\text{their } 0.943)$ or awrt 15.2. (Note area of $CBD = 15.177...$) A correct method for the area of triangle CBD which can be implied by awrt 15.2	M1
	$\pi - 1.4 - "\text{their } 0.943"$ A value for angle EBA of awrt 0.8 (from 0.7985926536... or 0.7983916536...) or value for angle EBA of $(1.74159... - \text{their angle } DBC)$ would imply this mark.		M1
	$AB = 5\cos(\pi - 1.4 - "0.943")$ or $AE = 5\sin(\pi - 1.4 - "0.943")$		
		$AB = 5\cos(\pi - 1.4 - \text{their } 0.943)$ $AB = 5\cos(0.79859...) = 3.488577938...$ Allow M1 for $AB =$ awrt 3.49 Or $AE = 5\sin(\pi - 1.4 - \text{their } 0.943)$ $AE = 5\sin(0.79859...) = 3.581874365688...$ Allow M1 for $AE =$ awrt 3.58 **It must be clear that** $\pi - 1.4 - "0.943"$ **is being used for angle EBA.** **Note that some candidates use the sin rule here but it must be used correctly – do not allow mixing of degrees and radians.**	M1
	Area $EAB = \frac{1}{2}5\cos(\pi - 1.4 - "0.943") \times 5\sin(\pi - 1.4 - "0.943")$		
	<u>This is dependent on the previous M1 and there must be no other errors in finding the area of triangle EAB</u>		**d**M1
	Allow M1 for area EAB = awrt 6.2		
	Area $ABCDE = 15.17... + 17.5 + 6.24... = 38.92...$		
		awrt 38.9	A1**cso**
			[5]
	Note that a sign error in (b) can give the obtuse angle (2.198....) and could lead to the correct answer in (c) – this would lose the final mark in (c)		**Total 9**

Question Number	Scheme		Marks
6(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $=160$	M1: Use of a correct S_∞ formula A1: 160	M1A1
	Accept correct answer only (160)		
			[2]
(b)	$S_{12} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}}$; $=127.77324...$	M1: Use of a correct S_n formula with $n = 12$ (condone missing brackets around 7/8) A1: **awrt** 127.8	M1A1
	T & I in (b) requires all 12 terms to be calculated correctly for M1 and A1 for awrt 127.8		
			[2]
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working. (condone missing brackets around 7/8)(Allow $=, <, >, \geq, \leq$) but see note below.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $+\left(\frac{7}{8}\right)^N$ oe (Allow $=, <, >, \geq, \leq$) but see note below. **Dependent on the previous M1**	**d**M1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the power law of logarithms or takes logs base 0.875 correctly to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ (Allow $=, <, >, \geq, \leq$) but see note below.	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but not $N > 44$	A1 **cso**
	An incorrect **<u>inequality</u>** statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			[4]
			Total 8
	<u>Trial & Improvement Method in (c):</u>		
	1st M1: Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		
	2nd M1: Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		
	3rd M1: For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		
	A1: $N = 44$ **cso**		
	Answer of N = 44 only with no working scores no marks		

Question Number	Scheme		Marks
7.	(i) $9\sin(\theta+60^\circ)=4$; $0\le\theta<360^\circ$ (ii) $2\tan x-3\sin x=0$; $-\pi\le x<\pi$		
(i)	$\sin(\theta+60^\circ)=\dfrac{4}{9}$, so $(\theta+60^\circ)=26.3877...$ $(\alpha=26.3877...)$	Sight of $\sin^{-1}\left(\dfrac{4}{9}\right)$ or awrt 26.4° or 0.461^{c} Can also be implied for $\theta=$ awrt -33.6 (i.e. $26.4-60$)	M1
	So, $\theta+60^\circ=\{153.6122...,\ 386.3877...\}$	$\theta+60^\circ$ = either "$180-$ their α" or "$360^\circ+$ their α" and not for $\theta=$ either "$180-$ their α" or "$360^\circ+$ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta=\{93.6122...,\ 326.3877...\}$	A1: At least one of awrt 93.6° or awrt 326.4° A1: Both awrt 93.6° and awrt 326.4°	A1 A1
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range. In an otherwise fully correct solution deduct the final A1for any extra solutions in range		
			[4]
(ii)	$2\left(\dfrac{\sin x}{\cos x}\right)-3\sin x=0$	Applies $\tan x=\dfrac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x=\dfrac{\sin x}{\cos x}$ can be implied by $2\tan x-3\sin x=0\Rightarrow\tan x(2-3\cos x)$		
	$2\sin x-3\sin x\cos x=0$		
	$\sin x(2-3\cos x)=0$		
	$\cos x=\dfrac{2}{3}$	$\cos x=\dfrac{2}{3}$	A1
	$x=$ awrt $\{0.84,\ -0.84\}$	A1: One of either awrt 0.84 or awrt -0.84 A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le1$	A1A1ft
	In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.		
	$\{\sin x=0\Rightarrow\}\ x=0$ **and** $-\pi$	**Both** $x=0$ **and** $-\pi$ or awrt -3.14 from $\sin x=0$ **In this part of the solution**, **ignore extra solutions in range.**	B1
	Note solutions are: $x=\{-3.1415...,\ -0.8410...,\ 0,\ 0.8410...\}$ Ignore extra solutions outside the range		
	For **all** answers in degrees in (ii) M1A1A0A1ftB0 is possible		
	Allow the use of θ in place of x in (ii)		
			[5]
			Total 9

Question Number	Scheme		Marks
8.	Graph of $y = 3^x$ and solving $3^{2x} - 9(3^x) + 18 = 0$		
(a)		At least two of the three criteria correct. **(See notes below.)**	B1
		All three criteria correct. **(See notes below.)**	B1
		Criteria number 1: Correct shape of curve for $x \geq 0$ and at least touches the positive y-axis. **Criteria number 2**: Correct shape of curve for $x < 0$. Must not touch the x-axis or have any turning points. **Criteria number 3**: $(0, 1)$ stated or in a table or 1 marked on the y-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the "correct" place on the y-axis.	
			[2]
(b)	$\left(3^x\right)^2 - 9\left(3^x\right) + 18 = 0$ or $y = 3^x \Rightarrow y^2 - 9y + 18 = 0$	Forms a quadratic of the correct form in 3^x or in "y" where $"y" = 3^x$ or even in x where $"x" = 3^x$	M1
	$\{ (y-6)(y-3) = 0$ or $\left(3^x - 6\right)\left(3^x - 3\right) = 0 \}$		
	$y = 6,\ y = 3$ or $3^x = 6,\ 3^x = 3$	**Both** $y = 6$ and $y = 3$.	A1
	$\left\{3^x = 6 \Rightarrow\right\}\ x\log 3 = \log 6$ or $x = \dfrac{\log 6}{\log 3}$ or $x = \log_3 6$	A valid method for solving $3^x = k$ where $k > 0,\ k \neq 1,\ k \neq 3$ **to give either** $\boxed{x\log 3 = \log k \text{ or } x = \dfrac{\log k}{\log 3} \text{ or } x = \log_3 k}$.	**d**M1
	$x = 1.63092...$	awrt 1.63	A1**cso**
	Provided the first M1A1 is scored, the second M1A1 can be implied by awrt 1.63		
	$x = 1$	$x = 1$ stated as a solution from ***any*** working.	B1
			[5]
			Total 7

Question Number	Scheme		Marks
	Mark (a) and (b) together		
9. (a)	$OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{=14\}$	Uses the addition form of Pythagoras on $6\sqrt{5}$ and 4. Condone missing brackets on $\left(6\sqrt{5}\right)^2$ **(Working or 14 may be seen on the diagram)**	M1
	$y_Q = \sqrt{14^2 - 11^2}$	$y_Q = \sqrt{(\text{their } OQ)^2 - 11^2}$ **Must include √ and is dependent on the first M1 and requires OQ > 11**	**d**M1
	$= \sqrt{75}$ or $5\sqrt{3}$	$\sqrt{75}$ or $5\sqrt{3}$	A1**cso**
			[3]
(b)	$(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$	M1: $(x \pm 11)^2 + (y \pm \text{their } k)^2 = 4^2$ **Equation must be of this form and must use *x* and *y* not other letters. *k* could be their last answer to part (a). Allow their *k* ≠ 0 or just the letter *k*.** A1: $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ or $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 4^2$ **NB** $5\sqrt{3}$ **must come from correct work in (a) and allow awrt 8.66**	M1A1
	Allow in expanded form for the final A1 e.g. $x^2 - 22x + 121 + y^2 - 10\sqrt{3}y + 75 = 16$		
			[2]
			Total 5
	Watch out for:		
	(a) $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2} = \sqrt{46}$ M1 $y_Q = \sqrt{46 - 11^2}$ M0 (OQ < 11) $y_Q = \sqrt{75}$ A0 (b) $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ M1A0		

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="2">10. (a)</td><td>$\frac{1}{2}(9x+6x)4x$
or $2x\times 15x$
or $\left(\frac{1}{2}4x\times(9x-6x)+6x\times 4x\right)$
or $6x^2+24x^2$
or $\left(9x\times 4x-\frac{1}{2}4x\times(9x-6x)\right)$
or $36x^2-6x^2$</td><td>M1: Correct attempt at the area of a trapezium.
Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x\times 6x$ is M0.
If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.</td><td rowspan="2">M1A1cso</td></tr>
<tr><td>$\Rightarrow 30x^2y=9600 \Rightarrow y=\frac{9600}{30x^2} \Rightarrow y=\frac{320}{x^2}$ *</td><td>A1: Correct proof with at least one intermediate step and no errors seen.
" y =" is required.</td></tr>
<tr><td></td><td colspan="2"></td><td>[2]</td></tr>
<tr><td rowspan="7">(b)</td><td colspan="2">$(S=)\ \frac{1}{2}(9x+6x)4x+\frac{1}{2}(9x+6x)4x+6xy+9xy+5xy+4xy$</td><td>M1A1</td></tr>
<tr><td colspan="2">M1: An attempt to find the area of six faces of the prism. The 2 trapezia may be combined as $(9x+6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct.
A1: Correct expression in any form.
Allow just $(S=)\ 60x^2+24xy$ for M1A1</td><td></td></tr>
<tr><td colspan="2">$y=\frac{320}{x^2} \Rightarrow (S=)\ 30x^2+30x^2+24x\left(\frac{320}{x^2}\right)$</td><td>M1</td></tr>
<tr><td colspan="2">Substitutes $y=\frac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.</td><td></td></tr>
<tr><td>So, $(S=)\ 60x^2+\frac{7680}{x}$ *</td><td>Correct solution only.
"S = " is not required here.</td><td>A1* cso</td></tr>
<tr><td colspan="2"></td><td>[4]</td></tr>
</table>

10(c)	$\frac{dS}{dx} = 120x - 7680x^{-2} \left\{ = 120x - \frac{7680}{x^2} \right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value" Setting their $\frac{dS}{dx} = 0$ and "candidate's ft ***correct*** power of x = a value". **The power of *x* must be consistent with their differentiation**. If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1**cso**
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find *S* – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880 \text{ (cm}^2)$	Substitute candidate's value of $x (\neq 0)$ into a formula for S. **Dependent on both previous M marks.**	**dd**M1
		2880 **cso (Must come from correct work)**	A1 **cao and cso**
			[6]

10(d)	$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt $S''\left(x^n \to x^{n-1}\right)$ **and** considers sign. This mark requires an attempt at the second derivative and **some consideration of its sign**. There does not necessarily need to be any substitution. An attempt to solve $S''=0$ is M0 A1: $120 + \frac{15360}{x^3}$ and >0 **and** conclusion. Requires a **correct** second derivative of $120 + \frac{15360}{x^3}$ (need not be simplified) **and** a valid reason (e.g. > 0), **and** conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been evaluated incorrectly.	M1A1ft
	A correct S'' followed by $S''("4")="360"$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''("4")="360"$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14